SH-III/Math-303/C-7/19

B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32113 **Course Code : SHMTH-303-C-7**

Course Title: Numerical Methods

Time: 1 Hour 15 Minutes **Full Marks: 25**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 1×5=5

(a) Determine the number of correct (significant) digits in the number $x = 0 \cdot 4785$ given its relative error $E_r = 0 \cdot 3 \times 10^{-2}$.

(b) Show that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$, where Δ is the forward difference operator.

(c) Explain why the degree of Precision of Simpson's one-third quadrature formula is 3.

(d) Write down the condition of convergence of the Newton-Raphson method for solving an equation $f(x) = 0$.

(e) In the algorithm of Runge-Kutta method of order 4, write the usual expressions for K_2 and K_3.

(f) Find y when $\frac{dy}{dx} = x + y^2$ with $y(0) = 0$ by Picard's approximation method after two iterations.

(g) Find the Lagrange's interpolation polynomial fitting the data points $f(1) = 6, f(3) = 0, f(4) = 12$ for some function $f(x)$.

(h) State the condition of convergence of Gauss-Seidal iteration method for solving numerically a system of linear algebraic equations.

2. Answer *any two* questions: 5×2=10

(a) Explain the Regula-Falsi method (method of False position) in obtaining a simple real root of an equation of the form $f(x) = 0$. Why does the method is called 'Linear interpolation method'? 4+1=5

(b) Prove that the remainder in approximating a function $f(x)$ by the interpolation polynomial $\phi(x)$ using interpolating points $x_0, x_1 \dots,\ x_n$ is of the form

$$(x-x_0)\,(x-x_1)\dots(x-x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}$$

where ξ lies between the smallest and the largest of the numbers $x, x_0, x_1 \dots x_n$

(c) Find by the Euler's modified method, the value of y for $x = 0\cdot05$ from the differential equation $\frac{dy}{dx} = x + y$. Correct up to four places of decimals with the initial condition $y = 1$ when $x = 0$.

(d) Using Simpson's one-third quadrature formula find the value of $\int_{1\cdot2}^{1\cdot6}\left(x+\frac{1}{x}\right)dx$; Correct up to two significant figures taking $n = 4$. Show the calculations side by side.

3. Answer *any one* question: 10×1=10

(a) (i) With an example illustrate the 'truncation error'.

(ii) Discuss the Geometrical significance of Trapezoidal rule.

(iii) With usual symbols, establish the relation $f[x_0, x_1 \dots, x_n] = \frac{\Delta^n y_0}{n!h^n}$ where $x_r = x_0 + rh,\ r = 1, 2, \dots, n.$ 2+3+5=10

(b) (i) Describe briefly Gauss elimination method for solving a system of Linear algebraic equations without pivoting.

(ii) Given $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$. Find $y(0\cdot1)$ by 4th order Runge-Kutta Method (Correct up to 4 decimal places). 5+5=10